Conformal fixed point, Cosmological Constant and Quintessence

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We connect a possible solution for the "cosmological constant problem" to the existence of a (postulated) conformal fixed point in a fundamental theory. The resulting cosmology leads to quintessence, where the present acceleration of the expansion of the universe is linked to a crossover in the flow of coupling constants.

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Once upon a time gravity was a strong force, with Newton's constant $G^{(i)}_{eff} = 10^{110} m^3 kg^{-1} s^{-2}$ or effective Planck mass $\bar{M}^{(i)} = 2 \cdot 10^{-33} eV$, and typical particle masses $\approx \bar{M}^{(i)}$. The mysterious homogeneous dark energy in the universe ("cosmological constant") started out with a similar characteristic magnitude $V^{(i)} \approx (\bar{M}^{(i)})^4$. Over the ages of the history of the universe the Planck mass increased and reaches today the value $\bar{M}^{(0)} = 2.44 \cdot 10^{18}\ GeV$. In the later stages of cosmology the mass ratios $M_W/\bar{M}$ and $m_p/\bar{M}$ for the Fermi scale and the proton mass have been approximately time independent. The growth rate of the dark energy was slower, however, such that today $V^{(0)} = (2.2 \cdot 10^{-3} eV)^4$, explaining one of the smallest numbers observed in nature, $V^{(0)}/\bar{M}^4 = 6.5 \cdot 10^{-121}$. In the present epoch the pace of change of the fundamental mass scales slows down considerably, resulting in an accelerated expansion of the universe. This tale of the cosmological history may seem somewhat weird at first sight - we will argue here that it could naturally be associated to the properties of a (postulated) conformal fixed point of a (still unknown) theory unifying all interactions.

Our basic assumption states that in a fundamental theory of all interactions (FT) all mass scales of particle physics are determined by a field χ rather than by a fundamental constant. This is common in grand unified, higher dimensional or superstring theories. Typically, χ is associated to a scale of transition such that for momenta $p^2 \gg \chi^2$ all the modes of the FT are important - for example, the FT may be formulated in more than four spacetime dimensions - whereas for $p^2 \ll \chi^2$ an effective description in terms of a four dimensional quantum field theory becomes valid. From the FT point of view the field χ plays the role of an effective infrared scale. Within the four dimensional description that we adopt here χ is a scalar field and may therefore evolve over cosmological time scales. In particular, the effective Planck mass is proportional to χ. If χ changes with time one is led to cosmologies with a variable Planck mass [1].

Dilatation or scale transformations correspond to a multiplicative rescaling $\chi \to c\chi$, with constant c and appropriate scaling of the metric and other fields. If dilatation symmetry were an exact symmetry the value of χ would not be an observable quantity. However, in quantum theories it is common that dilatation symmetry is violated * by the effects of fluctuations, resulting in "running" dimensionless couplings depending on χ. By dimensional transmutation this introduces an intrinsic scale m, in anlogy to the characteristic scale of strong interactions, Λ_{QCD}. Our assumption means more precisely that m is the *only* intrinsic mass scale of the FT, such that for $\chi \gg m$ the world looks effectively dilatation symmetric, up to small corrections that vanish for $m/\chi \to 0$.

As an example, we realize these ideas in an effective model for gravity and the cosmon field χ, characterized by an effective action S after "integrating out" the other fields and all quantum fluctuations

$$S = \int d^4x \sqrt{g} \Big\{ -\frac{1}{2}\chi^2 R + \frac{1}{2}\Big(\delta\left(\frac{\chi}{m}\right) - 6\Big) \partial^\mu\chi\partial_\mu\chi + V(\chi)\Big\}. \quad (1)$$

Here we consider the case where for $m \to 0$ the effective potential has a flat direction, and assume that for large χ the leading manifestation of the dilatation anomaly results in a mass term

$$V = m^2\chi^2. \quad (2)$$

In the region of large $\chi \gg m$ all interactions are derivative interactions. The dimesionless coupling $\delta > 0$ governs the cosmon kinetic term. The additive constant is chosen such that the model exhibits an exact local conformal symmetry $g_{\mu\nu} \to c^{-2}(x) g_{\mu\nu}, \chi \to c(x)\chi$ for $\delta = 0$, $m = 0$. In our normalization χ corresponds to the effective reduced Planck mass, $\bar{M} = (8\pi G_{eff})^{-1/2}$.

It is straightforward to solve the field equations for this model [1] [4] for a homogeneous and isotropic metric and cosmon field. One finds that the cosmon field increases for large time! This is due to its coupling to gravity and contradicts the too naive expectation that the cosmon should approach the potential minimum at $\chi = 0$. The increase of χ has a striking consequence for the fate of the homogeneous dark energy. Indeed, only mass ratios are physically observable [1]. For the effective "cosmological constant" $V/\bar{M}^4$ we find that $V^{1/4}$

*For details of the formalism see [2] [3].

increases less rapidly than $\bar{M} = \chi$. Therefore the dark energy vanishes asymptotically if χ increases with time, i. e. $V/\bar{M}^4 = m^2/\chi^2 \to 0$. This is the basic ingredient for our explanation [3] why the "cosmological constant" vanishes asymptotically and why dark energy has attained an extremely small value today as a consequence of the enormous age of our universe.

Before we can proceed to a quantitative discussion of cosmology we need to specify $\delta(\chi/m)$. For large $\chi \gg m$ simple dimensional arguments tell us that the χ-dependence can be written in terms of a "renormalization group equation"

$$\frac{\partial \delta}{\partial \ln \chi} = \beta_\delta(\delta). \tag{3}$$

A computation of β_δ would need the knowledge of the FT since the dominant contributions arise from modes with $p^2 \approx \chi^2$. We only know that β_δ should have a zero for $\delta = 0$, since $\delta = 0$ corresponds to an enhanced (conformal) symmetry and separates a stable model for $\delta \geq 0$ from an unacceptable unstable model for $\delta < 0$. By continuity, for small enough δ the β- function is also small and δ increases only slowly with χ (assuming $\beta_\delta \geq 0$). For χ varying over many orders of magnitude during the cosmological evolution we may nevertheless be confronted with a situation where δ has grown large at some critical value χ_c. At this scale we expect a *crossover* from the vicinity of the conformal fixed point at $\delta = 0$ to an unknown behavior for large δ. The crossover scale χ_c can play an important role in cosmology. In particular, we will discuss a scenario where χ reaches χ_c in the present epoch, triggering an accelerated expansion of the universe [5].

As a simple example we take

$$\beta_\delta = E\delta^2 \ , \ \delta = \frac{1}{E\ln(\chi_c/\chi)} \ , \tag{4}$$

where χ_c depends on E and the "initial value" $\delta_i = \delta(\chi = m)$. For small $E\delta_i$ the separation between the crossover scale χ_c and the intrinsic scale m becomes exponentially large

$$\frac{\chi_c}{m} = \exp\left(\frac{1}{E\delta_i}\right) \ , \tag{5}$$

in close analogy to the inverse ratio between the strong interaction scale Λ_{QCD} and the unification scale. In particular, if χ_c is associated to the present value $\bar{M}^{(0)} = \bar{M}_p = 2.44 \cdot 10^{18} GeV$ and $E\delta_i \approx 1/138$ we obtain a present value for the potential part of the dark energy

$$\begin{aligned} V^{(0)} &= m^2\chi_c^2 = \exp\left(-\frac{2}{E\delta_i}\right)\bar{M}_p^4 \\ &= 6.56 \cdot 10^{-121}\bar{M}_p^4 = (2.2 \cdot 10^{-3} eV)^4. \end{aligned} \tag{6}$$

We emphasize that the cosmological evolution for this class of models is independent of the initial conditions since the late time behavior is governed by a cosmic attractor solution [3] [6] [7]. Generically, the ratio Ω_h between the homogeneous dark energy density and the critical energy density stays small as long as δ is small, adjusting itself to a dominant radiation or matter component, $\Omega_h \approx \delta$ or $\Omega_h \approx \frac{3}{4}\delta$, respectively. This behavior only changes once χ reaches the crossover scale χ_c, and for $E\delta_i \approx 1/138$ this happens precisely at the present epoch. Then the universe switches to a regime where dark energy dominates.

To be more quantitative we select $E = 2.5$, $\delta_i = 2.9 \cdot 10^{-3}$. We can now compute the characteristic quantities like the amount of dark energy today, $\Omega_h^{(0)} = 0.75$, or the equation of state $w_h = p_h/\rho_h$ at the present time, $w_h^{(0)} = -0.87$. They are compatible with the supernovae observations [5] and the age of the universe $t^{(0)} = 14.2 \cdot 10^9 yr$. For a discussion [8] of the spectrum of the CMB-unisotropies we need, in addition, the value of $\Omega_h^{(ls)} = 0.038$ at the time of last scattering and an averaged equation of state $\bar{w}$ which determine the position of the third peak in angular momentum space as $l_3 = 806$ (for $h = 0.65$). Structure formation is slowed down by the early presence of dark energy [9]. It depends on an average of Ω_h over the time of structure formation, $\Omega_h^{(sf)} = 0.07$ [10]. In our case the density fluctuations are reduced by a factor $\sigma_8/\sigma_8(\Lambda) = 0.54$ as compared to a model with a cosmological constant and the same amount of dark energy today. We conclude that our simple model is compatible with the present observations. We observe interesting differences as compared to models with a cosmological constant. They are subject to future observational tests.

Several comments are in order: (i) Cosmology is most easily discussed after a Weyl scaling $g_{\mu\nu} \to (\bar{M}_p/\chi)^2 g_{\mu\nu}$ and a redefinition of the cosmon field $\varphi/\bar{M}_p = \ln\left(\chi^4/V(\chi)\right) = 2\ln(\chi/m)$, such that the coefficient in front of the curvature scalar R becomes constant and φ is directly related to the value of the potential

$$\begin{aligned} S = \int d^4x\sqrt{g}\Big\{ &-\frac{1}{2}\bar{M}_p^2 R \\ &+\frac{1}{2}k^2(\varphi)\partial^\mu\varphi\partial_\mu\varphi + \bar{M}_p^4 \exp\left(-\frac{\varphi}{\bar{M}_p}\right)\Big\}. \end{aligned} \tag{7}$$

The details of the model are now encoded † in the nontrivial kinetic term [11]

$$k^2(\varphi) = \frac{1}{4}\delta = \frac{\bar{M}_p}{2E(\varphi_c - \varphi)}. \tag{8}$$

†Note that the scale m does not appear anymore in eq. (8). Indeed, m can be scaled to an arbitrary value by a multiplicative rescaling of χ and $g_{\mu\nu}$ which corresponds, after Weyl scaling, to a shift in φ.

Here the singularity at $\varphi_c/\bar{M}_p = 2/(\delta_i E)$ is a particular feature of eq. (4) and does not occur if δ remains finite. As a general feature, the motion of the cosmon slows down once k^2 becomes large such that the dominance of the potential V over the kinetic energy T leads to a negative equation of state $w_h = (T-V)/(T+V)$ and to an acceleration of the universe [12].

ii) The qualitative features of our proposal hold for a much more general class of cosmon potentials V. For example, adding to eq. (2) a "bare cosmological constant" γm^4 becomes completely irrelevant for large χ/m. Only the behavior of V for large χ matters. The scenario of an asymptotically vanishing dark energy holds provided that V increases less rapidly than χ^4 and δ remains finite for $\chi < \bar{M}_p$. This applies, in particular, to an asymptotic behavior $V \to$ const or to $V = \lambda(\chi/m)\chi^4$ with a dimensionless coupling λ obeying the renormalization group equation ‡

$$\frac{\partial \lambda}{\partial \ln \chi} = -A\lambda \ , \ A > 0. \tag{9}$$

iii) Details of cosmology depend on β_δ. For $\beta_\delta = 0$ one recovers "exponential quintessence" [3], whereas for $\beta_\delta = D\delta$, D constant, one finds "inverse power law quintessence" [6] with power $\alpha = 2A/D$. We have studied other models with crossover behavior, e. g.: $\beta_\delta = D\delta + E\delta^2$. For $D > 0$ the required value of δ_i decreases and early quintessence (e. g. $\Omega_h^{(ls)}$, $\Omega_h^{(sf)}$) becomes less important. The precise flow for very large δ, i. e. δ remaining finite for all χ, plays only a minor role for presently observable cosmology provided δ grows sufficiently large in the present epoch.

iv) We can extend our description to matter fields and radiation. As an example we consider the Higgs doublet H, a fermion field ψ and the gluons characterized by their field strength $F_{\mu\nu}$. Within our assumption this adds to eq. (1) a term

$$\begin{aligned} S_M = \int d^4x\sqrt{g}\Big\{&(\lambda_H/2)(H^\dagger H - \beta^2\chi^2)^2 \\ &+(h\bar{\psi}_L H\psi_R + h.c.) + \frac{Z_F}{4}F^{\mu\nu}F_{\mu\nu}\Big\}. \end{aligned} \tag{10}$$

Our previous description has neglected the possible χ-dependence of the dimensionless couplings λ_H, β, h, Z_F such that after the Weyl scaling φ decouples completely from matter and radiation. In this case the Higgs doublet reaches its χ- dependent minimum $|H|^2 = \beta^2\chi^2$ early in cosmology (after the electroweak phase transition). Similarly, for a fixed value of the running gauge coupling at some grand unified scale M_X , $\alpha_S(M_X) \approx 1/40$, we find $\Lambda_{QCD} \sim \chi$ if $M_X \sim \chi$. In this approximation all ratios of particle masses become independent of χ and do not vary with cosmological time.

We note the appearance of two different types of characteristic masses for the excitations. The excitation along the "vacuum direction" corresponds to a simultaneous change of *all* mass scales (along the direction $|H| = \beta\chi$). Its mass is given by the small intrinsic mass m. On the other hand, the excitations perpendicular to the "vacuum direction" correspond to a variation of mass ratios and have a characteristic mass $\sim \chi$. In our example, the intrinsic mass m is many orders of magnitude smaller than the (present) mass of the Higgs boson $M_H^2 = \lambda_H\beta^2\chi^2$. This resembles the well known case of spontaneous breaking of a global symmetry at the scale χ. In this analogy a small mass m for the pseudo- Goldstone boson is induced by an anomaly or explicit symmetry violation.

v) We do not expect the dimensionless couplings λ_H, β, h, Z_F to be precisely independent of χ/m. Then the cosmological variation of χ/m will induce a time dependence of the fundamental parameters. Severe bounds [13] restrict [1] [14] this dependence for the dimensionless couplings of the known fields. More freedom is left for a coupling of the cosmon φ to dark matter- sizeable couplings would influence the cosmology [4] [15]. Very close to the big bang, for $\chi \approx m$, the dependence of all couplings on χ/m may have been strong.

vi) In a grand unified theory the renormalized strong gauge coupling or the fine structure constant α_{em} depend on the value of the gauge coupling $g_X = g(M_X)$ at the unification scale M_X where $g_X^2(\chi) \sim Z_F^{-1}(\chi)$. We neglect § here for simplicity the χ-dependence of M_X , $B_X = -\partial \ln(M_X/\chi)/\partial \ln \chi \approx 0$, and concentrate on the case where for $\chi \to \infty$ the running of g_X is governed by a fixed point $g_*^2/4\pi \approx 1/40$

$$\begin{aligned} \frac{\partial g_X^2}{\partial \ln \chi} = \beta_{g^2} = B_2 g_X^2 - B_4 g_X^4 \ , \\ B_2 = B_4 g_*^2 > 0. \end{aligned} \tag{11}$$

It is interesting to associate m with the nonperturbative scale where $g_X(\chi \to m) \to \infty$,

$$g_X^2 = g_*^2\left[1 - \left(\frac{\chi}{m}\right)^{-B_2}\right]^{-1}. \tag{12}$$

The present relative variations of the gauge couplings are then determined by

$$\eta_F = -\frac{\beta_g^2}{g_X^2} \approx B_2\left(\frac{\chi}{m}\right)^{-B_2} = \exp\left(-\frac{B_2\varphi}{2\bar{M}_p}\right). \tag{13}$$

For sufficiently large B_2, say $B_2 > 0.2$, the time variation of Λ_{QCD} or α_{em} is much too small to be accessible for

‡The potential (2) corresponds to $A = 2$. A constant value $A \neq 2$ could be absorbed by a rescaling $\ln \chi \to (2/A)\ln \chi$ and rescaling of $g_{\mu\nu}$. This modifies the relation (8) to $k^2 = \delta/A^2$.

§See [14] for an extended discussion.

present observations [14]. More generally, we conclude that a fixed point which is approached sufficiently fast for $\chi \to \infty$ could give a very simple explanation why the cosmological time variation of fundamental couplings is small. On the other hand, the substantial variation of δ at the present cosmological epoch may have a small influence on the precise location of the fixed point $g_*(\delta)$. It is well conceivable that a small increase of $g_*(\delta)$ could lead to a time variation of α_{em} in the range inferred from the observation of quasar absorption lines [16], corresponding to $\eta_F = -4 \cdot 10^{-7}$.

vii) The effect of the quantum fluctuations is encoded in the β-functions (3), (9), (11). In our setting this concerns mainly small deviations from a conformal fixed point at $\delta = 0$, $\lambda = 0$, $g^2 = g_*^2$. We emphasize that the existence of a fixed point at $\lambda = 0$ seems plausible since it separates again a stable ($\lambda > 0$) from an unstable ($\lambda < 0$, unbounded potential) situation.

The conformal fixed point has λ and g^2 as relevant couplings for $\chi \to 0$ whereas for $\chi \to \infty$ the relevant coupling corresponds to δ. The flow of the couplings is therefore neither stable towards the infrared nor towards the ultraviolet. The scale m marks the first infrared scale where couplings grow large (e. g. the gauge coupling) whereas χ_c corresponds to the instability in the ultraviolet. A huge ratio χ_c/m occurs whenever the trajectory of the flow passes sufficiently close to the fixed point.

It may be argued that for all χ the value of the potential V should be of the same size as the contribution from the quantum fluctuations (vacuum energy) of some individual particle species (e. g. electrons or photons) in the momentum range $p^2 < \chi^2$, leading to $V \sim \chi^4$. This naive guess seems to us quite misleading in a situation where the fluctuations with $p^2 \approx \chi^2$ of all modes of the FT dominate. Such a guess would be completely unjustified for the behavior near (partial) fixed points which characterize critical behavior in statistical physics.

In conclusion, the existence of a conformal fixed point in a fundamental theory together with the flow of small deviations from the fixed point proposed in this note ($A > 0$) would lead to a natural explanation why the cosmological constant vanishes for asymptotic time. After Weyl scaling, no additive constant hinders the asymptotic approach of the cosmon potential to zero, $V(\varphi \to \infty) \to 0$. This would solve the "cosmological constant problem" [17]. In our crossover scenario the past evolution of the universe is characterized by a small and slowly varying fraction of dark energy which adapts to a dominant radiation (matter) component $\Omega_h \approx \delta \left(\frac{3}{4}\delta\right)$. The future of the universe depends crucially on the unknown properties of the flow of $\delta(\chi)$ in the region of the large δ. The present epoch witnesses a crossover from small to large δ, resulting in an accelerated expansion. In a FT the crossover scale χ_c/m should be computable, just as the values of mass ratios or dimensionless couplings in particle physics. It is therefore not excluded that some of the "cosmic coincidences" (relations between the present value of the Hubble parameter H_0 and particle properties) could find an explanation in this direction. For the potential (2) the present value of H is given by the mass m characterizing the dilatation anomaly

$$H_0^2 = \frac{2\Omega_h^{(0)}}{3(1-w_h^{(0)})} m^2. \tag{14}$$

The time variation of the dark energy could be detected by cosmological observations in the near feature, and an establishment of a time variation of fundamental couplings would be a striking argument in favor of our proposal.

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